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Downside Variance Risk Premium

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Downside Variance Risk Premium

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Abstract

We propose a new decomposition of the variance risk premium (VRP) in terms of upside and downside variance risk premia. These components reflect market compensation for changes in good and bad uncertainties. Their difference is a measure of skewness risk premium, which captures investors' asymmetric views on favorable versus undesirable risks. Empirically, we establish that the downside variance risk premium is the main component of the VRP. We find a positive and significant link between the downside variance risk premium and the equity premium, as well as a negative and significant relation between the skewness risk premium and the equity premium. Moreover, we document that the skewness risk premium fills the time gap between one-quarter-ahead returns predictability, delivered by the variance risk premium, and long-term predictors such as price-dividend or price-earning ratios. A simple equilibrium consumption-based asset pricing model, fitted to the U.S. data, supports our decomposition.

Keywords: Risk-neutral volatility, Realized volatility, Downside and upside variance risk premium, Skewness risk premium

JEL Classification: C58, G12

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1 Introduction

A fundamental relationship in asset pricing posits a positive relation between risk and asset returns, see Merton (1973). Empirically, measures of risk based on historical asset prices lead to inconclusive estimates of this relation.[1] Two recent strands of asset pricing literature have success in addressing these conflicting results caused by using historical measures of risk in empirical exercises. In one line of research, several studies have documented the asymmetry of risk-return trade off in response to negative and positive realizations in the financial markets. Among them, studies by Bonomo, Garcia, Meddahi, and Tédongap (2011), Feunou, Jahan-Parvar, and Tédongap (2013), and Rossi and Timmermann (2015) are related to our work. In particular, Feunou, Jahan-Parvar, and Tédongap (2013) explicitly model the upside and downside volatilities (the risk borne by market participants if realized returns exceed or fall below a certain threshold), document the impact of asymmetries on risk-return trade off and highlight the role of downside risk. They develop a methodology that delivers an empirically robust, positive relation between risk and returns by allowing a time-varying market price of risk and asymmetric return distributions. In a second line of research, studies such as Bakshi and Kapadia (2003), Carr and Wu (2009) and Bollerslev, Tauchen, and Zhou (2009) rely on the information in option prices to measure time-varying risk compensations in the data. In particular, Bollerslev, Tauchen, and Zhou (2009) (henceforth, BTZ) study the variance premium (henceforth, VRP), defined as the difference between the risk-neutral and physical expectations of returns variation. The VRP, as formalized and studied by BTZ, is a robust predictor of asset returns at maturities of 3 to 6 months. Because of its significant predictive power for short-term asset returns, the VRP is often viewed as reflecting investors' appraisal of changes in near future volatility.

In this paper, we bridge the gap between these two strands of the literature. We propose a new decomposition of the VRP in terms of upside and downside variance risk premia (VRP^U and VRP^D, respectively).[2] We subsequently uncover a common component shared by the VRP and

[1]Among many studies, Brandt and Kang (2004) observe a negative relation between realized market risk and returns. Ghysels, Santa-Clara, and Valkanov (2005) and Ludvigson and Ng (2007) find a positive relation. On the other hand, Baillie and DeGennaro (1990) and Bollerslev and Zhou (2006) document mixed results.

[2]We define the down(up)side variance as the realized variance of the stock market returns for negative (positive) returns. The down(up)side variance risk premium is the difference between option-implied and realized down(up)side variance. Decomposing variance in this way was pioneered by Barndorff-Nielsen, Kinnebrock, and Shephard (2010). We define the difference between upside and downside variances as the relative upside variance. Feunou, Jahan-Parvar,

the skewness risk premium (henceforth, SRP), which is the VRP^D. We show that this component is the basis for many empirical regularities in aggregate market returns uncovered by recent studies and document novel theoretical and empirical findings.

Intuitively, this work is motivated by a simple observation: Investors like good uncertainty as it increases the potential of substantial gains, but dislike bad uncertainty, as it increases the likelihood of severe losses. We link our empirical results to consumption-based asset pricing models that feature the so-called "good environment – bad environment" structure. We provide theoretical support for our empirical findings through a structural model that shares similarities with models developed by Segal, Shaliastovich, and Yaron (2015), and Bekaert and Engstrom (2015). We define "good uncertainty" and "bad uncertainty" as volatility associated with positive or negative shocks to fundamentals such as dividend and consumption growth. Bekaert and Engstrom (2015) consider a "good environment" (resp. "bad environment") economy where positively (resp. negatively) skewed innovations dominate negatively (resp. positively) skewed shocks to consumption growth. Given that investors tend to hedge against downward movements to avoid losses, the VRP^D is expected to be generally positive-valued and the main driver of the VRP. Conversely, investors often find upside movements desirable. They are willing to pay for exposure to such movements and the potential for higher profits. Thus, we expect a mostly negative-valued VRP^U. Theoretically, we support our empirical findings with a simple endowment equilibrium asset pricing model, where the representative agent is endowed with Epstein and Zin (1989) preferences, and where the consumption growth process is affected by distinct upside and downside shocks. Our model shares some features with Bansal and Yaron (2004), BTZ, and Segal, Shaliastovich, and Yaron (2015), among others. We fit this model to data, and compute the model-implied equity, upside variance, downside variance, and skewness risk premia (analytically derived in closed form) that support the model free, high frequency data-based empirical findings reported in the paper.

This study highlights the importance of asymmetry in the assessment of risk. As mentioned previously, we find VRP^D to be (generally) positive (reflecting the compensation required by an investor to bear the downside risk), whereas VRP^U is (generally) negative (as it is the discount conceded by an investor to secure exposure to such shocks). Thus, the (total) variance risk premium

and Tédongap (2016) show that relative upside variance is a measure of skewness. Based on their work, we use the difference between option-implied and realized relative upside variances as a measure of the skewness risk premium.

that sums these two components lumps together market participants' (asymmetric) views about good and bad uncertainties. As a result, a positive (total) variance risk premium reflects the investors willingness to pay more in order to hedge against changes in bad uncertainty than for exposure to variations in good uncertainty. Hence, focusing on the (total) variance risk premium does not provide a clear view of the trade-off between good and bad uncertainties, as a small positive VRP quantity does not necessarily imply a lower level of risk and/or risk aversion. Rather, it is an indication of a smaller difference between what agents are willing to pay for downside variation hedging versus upside variation exposure.

Our findings imply that, while the VRP^D explains the empirical regularities reported by BTZ (including the hump-shaped R^2 and slope parameter patterns), the VRP^U's contribution to the results reported by BTZ is more subdued. Additionally, we document the contribution of the difference between VRP^D and VRP^U (which is a proxy for skewness risk premium, SRP), to the predictability of returns that takes effect beyond the one-quarter-ahead window documented by BTZ. We find that the prediction power of VRP^D and SRP proxy increase over the term structure of equity returns. In addition, through extensive robustness testing, we establish that our findings are robust to the inclusion of a wide variety of common equity risk premium predictors. This leads to the conclusion that the predictability of aggregate returns by downside risk and skewness measures introduced here is independent from other common pricing ratios, such as the price-dividend ratio, price-earnings ratio, or default spread. Thus, we are able to close the horizon gap between short-term models such as BTZ and long-horizon predictive models such as Fama and French (1988), Campbell and Shiller (1988), Cochrane (1991), and Lettau and Ludvigson (2001).

1.1 Related literature

This paper is related to the mounting literature on the properties of the VRP, as discussed in earlier works by Bakshi and Kapadia (2003), Vilkov (2008), and Carr and Wu (2009), among others. Theoretical attempts to rationalize the observed dynamics of the VRP have led to both reduced-form and general equilibrium models in the literature. Within the reduced-form framework, Todorov (2010) focuses on the temporal dependence of continuous versus discontinuous VRP components driven by a semiparametric stochastic volatility model. He documents that both components exhibit nontrivial dynamics driven by *ex ante* volatility changes over time, coupled with *unanticipated* extreme swings in the market. In a general equilibrium setting, BTZ design a simple model in which time-varying volatility-of-volatility of consumption growth is the key determinant of the VRP. Drechsler and Yaron (2011) provide an equilibrium specification that features long-run risks and discontinuities in the stochastic volatility process governing the level of uncertainty about the cash-flow process. They extend the model of Bansal and Yaron (2004) by introducing a compound Poisson jump process in the state variable specification, thus departing from BTZ's assumption of Gaussian economic shocks. Our theoretical framework also extends BTZ's model, as we specify asymmetric predictable consumption growth components and differences of centered Gamma shocks to fundamentals.

Another strand of the literature explores the explanatory ability of the VRP. Along the time-series dimension, BTZ, Drechsler and Yaron (2011), and Kelly and Jiang (2014), among others, show that the VRP can help forecast the temporal variation in the aggregate stock market returns with high (low) premia predicting high (low) future returns, especially in within-the-year time scale. Ang, Hodrick, Xing, and Zhang (2006), and Cremers, Halling, and Weinbaum (2015), among others, find that the price of variance risk successfully explains a large set of expected stock returns in the cross-section of assets.

Drawing on existing decompositions of the quadratic variation of stock returns, other studies investigate the sources of variation in the VRP. Bollerslev and Todorov (2011) assess the importance of the premium related to extreme rare events by decomposing the VRP in terms of the diffusive and jump risk compensations. The authors show that the contribution of the jump tail risk premium is sizeable, and propose a new index to assess the compensation for concerns about disastrous outcomes – when returns (r_t) fall below a given threshold ($-\kappa$). In the Bollerslev and Todorov (2011) setting, κ is a strictly positive threshold that separates small (diffusion) from large (jump) variations. Due to the lack of liquidity for deep out-of-the money options, implementation of the jump tail risk premium approach for large values of κ is challenging. In practice, this procedure necessitates an additional extreme value theory (EVT) approximation step to extrapolate tail densities, especially under the risk-neutral measure. Our decomposition (based on a threshold of $\kappa = 0$) is much simpler to implement and interpret, as it does not require any explicit model for describing the tail behavior of the distributions underlying various premia.

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) propose a different decomposition of the realized variance in terms of upside and downside semi-variances obtained by summing high-frequency positive and negative squared returns, respectively. Other authors have used the same decomposition of the realized variance with either a focus on realized variance predictability (Patton and Sheppard, 2015), or on equity risk premium predictability (Guo, Wang, and Zhou, 2015). These studies focus exclusively on realized measures and do not use option prices to infer the risk-neutral counterparts and deduce the corresponding premia. In comparison, our work clearly evaluates the premia associated with upside and downside semi-variances, both realized and risk-neutral. In a related study, Kilic and Shaliastovich (2015) study a decomposition of the variance premium into the components associated with good and bad "events," modeled as jump processes.

Other related studies aim at decomposing the variance of macroeconomic variables. Feunou, Jahan-Parvar, and Tédongap (2013) study the dynamics of risk-return tradeoff in an equilibrium model where a representative agent endowed with disappointment aversion preferences of Gul (1991), faces asymmetric, bi-normal shocks to risky asset returns. Bekaert and Engstrom (2015) append the external habit formation model with "bad environment-good environment" dynamics driven by a convolution of positively and negatively skewed shocks to consumption and dividend growth. Segal, Shaliastovich, and Yaron (2015) study the impact of changes in *good* versus *bad* uncertainty on aggregate consumption growth and asset values. These authors demonstrate that these different types of uncertainties have opposite effects, with *good* (*bad*) economic risk implying a rise (decline) in future wealth or consumption growth. They characterize the role of asymmetric uncertainties in the determination of the economic activity level. Similar to these studies, we develop and estimate a consumption based equilibrium asset pricing model to highlight the roles that upside and downside variances play in pricing a risky asset in an otherwise standard asset pricing model.

Our study comprises two natural and linked components. First, we study the inherent asymmetry in responses of market participants to negative and positive market outcomes. To accomplish this goal, we draw on the vast existing literature on realized and risk-neutral volatility measures and their properties to construct nonparametric measures of up and down realized and risk-neutral semi-variances. We then show empirically how the stylized facts documented in the VRP literature are driven almost entirely by the contribution of VRP^D. As in Chang, Christoffersen, and Jacobs (2013), our approach avoids the traditional trade-off problem with estimates of higher moments from historical returns data needing long windows to increase precision but short windows to obtain conditional – instead of unconditional – estimates. As a byproduct, we show that using the relative upside variance, a nonparametric measure of skewness, we can enhance the predictive power of the variance risk premium to horizons beyond one quarter ahead. Second, we link the regularities documented in our empirical study to our equilibrium model.

Thus, we need reliable measures for realized and risk-neutral variance and skewness. A sizeable portion of empirical finance and financial econometrics literature is devoted to measures of volatility. Canonical papers focused on the properties and construction of realized volatility include Andersen, Bollerslev, Diebold, and Ebens (2001a); Andersen, Bollerslev, Diebold, and Labys (2001b); and Andersen, Bollerslev, Diebold, and Labys (2003), among others. The construction of realized downside and upside volatilities (also known as realized semi-variances) is addressed in Barndorff-Nielsen, Kinnebrock, and Shephard (2010). We follow the consensus in the literature about the construction of these measures. Similarly, based on pioneering studies such as Carr and Madan (1998, 1999, 2001) and Bakshi, Kapadia, and Madan (2003), we have a clear view on how to construct risk-neutral measures of volatility. The construction of option-implied downside and upside volatilities is addressed in Andersen, Bondarenko, and Gonzalez-Perez (2015). Again, we follow the existing literature in this respect.

In our study, we examine the contribution of realized and risk-neutral asymmetry. Presence of asymmetry in physical and risk-neutral distributions gives rise to skewness risk premium, SRP, which captures the wedge between the objective and the risk-neutral expectation of a realized skewness measure. Traditional measures of skewness have well-documented empirical problems. Kim and White (2004) demonstrate the limitations of estimating the traditional third moment. Harvey and Siddique (1999, 2000) explore time variation in conditional skewness by imposing autoregressive structures. More recently, Feunou, Jahan-Parvar, and Tédongap (2013) and Ghysels, Plazzi, and Valkanov (2016) use robust Pearson and Bowley's skewness measures to overcome many problems associated with the centered third moment, such as the excessive sensitivity to outliers documented in Kim and White (2004).

Amaya, Christoffersen, Jacobs, and Vasquez (2015) and Chang, Christoffersen, and Jacobs (2013) use measures similar to Neuberger's 2012 realized skewness in predicting the cross-section of returns at a weekly frequency. While Colacito, Ghysels, Meng, and Siwasarit (2016) study the predictive relationship between physical third cross-sectional moment of the distribution of GDP growth rates made by professional forecasters and equity excess returns, Conrad, Dittmar, and Ghysels (2013) investigate whether a risk-neutral measure of skewness has predictive power for the cross-section of returns. Our realized measure of asymmetry is simply the difference between the upside and the downside realized semi-variance, which turns out to be the so-called *signed jump variation* introduced in Patton and Sheppard (2015). Patton and Sheppard (2015) demonstrate that *signed jumps* improve the prediction of future realized variance.

Traditional option-based estimates of asymmetry, as Dennis and Mayhew (2009) show, are noisy. Our proposed risk-neutral skewness measure, in contrast, is well-behaved, easy to build, and easy to interpret. Kozhan, Neuberger, and Schneider (2014) propose an alternative approach to construct the skewness risk premium from cubic swap contracts. In line with our results, Kozhan et al. find a strong link between the VRP and the SRP. They consider contemporaneous correlations between variance and skewness premia and market excess returns. In contrast, we study the predictability of market returns by VRP and its components, in time-horizons ranging between one month to one year ahead. In addition, we refine their finding of a common factor between VRP and SRP, by showing that this factor is shared between VRP^D and SRP. As such, our findings yield a clear link between downside risk, skewness and left-tail outcomes.

The rest of the paper proceeds as follows. In Section 2, we present our decomposition of the VRP and the method for construction of risk-neutral and realized semi-variances. Section 3 details the data used in this study and the empirical construction of predictive variables used in our analysis. We present and discuss our main empirical results in Section 4. Specifically, we intuitively describe the components of variance risk premia, discuss their predictive ability, and explore the robustness of our findings. In Section 5, we introduce and estimate a simple equilibrium consumption-based asset pricing model that supports our empirical results. Section 6 concludes.

2 Decomposition of the variance risk premium

In what follows, we decompose equity price changes into positive and negative returns with respect to a suitably chosen threshold. In this study, we set this threshold to zero, but it can assume other values, given the questions to be answered. We sequentially build measures for upside and downside variances and for skewness. When taken to data, these measures are constructed non-parametrically.

We posit that equity market indices such as the S&P 500, S, are defined over the physical probability space characterized by $(\Omega, \mathbb{P}, \mathcal{F})$, where $\{\mathcal{F}_t\}_{t=0}^{\infty} \in \mathcal{F}$ are progressive filters on $\mathcal{F}$. The risk-neutral probability measure $\mathbb{Q}$ is related to the physical measure $\mathbb{P}$ through Girsanov's change of measure relation $\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_T} = Z_T, T < \infty$. At time t, we denote total equity returns as $R_t^e = (S_t + D_t - S_{t-1})/S_{t-1}$, where D_t is the dividend paid out in period $[t-1, t]$. In high-enough sampling frequencies, D_t is equal to zero. Then, we denote the log of prices by $s_t = \ln S_t$, log-returns by $r_t = s_t - s_{t-1}$, and excess log-returns by $r_t^e = r_t - r_t^f$, where r_t^f is the risk-free rate observed at time $t-1$. We obtain cumulative excess returns by summing one-period excess returns, $r_{t \to t+k}^e = \sum_{j=0}^{k} r_{t+j}^e$, where k is our prediction horizon.

We build the variance risk premium components following the steps in BTZ as the difference between option-implied and realized variances. Alternatively, these two components could be viewed as variances under risk-neutral and physical measures, respectively. In our approach, this construction requires three distinct steps: building the upside and downside realized variances, computing their expectations under the physical measure, and then doing the same under the risk-neutral measure.

2.1 Construction of the realized variance components

Following Andersen et al. (2003, 2001a), we construct the realized variance of returns on any given trading day t as $RV_t = \sum_{j=1}^{n_t} r_{j,t}^2$, where $r_{j,t}^2$ is the j^{th} intraday squared log-return and n_t is the number of intraday returns recorded on that day. We add the squared overnight log-return (the difference in log price between when the market opens at t and when it closes at $t-1$), and we scale the RV_t series to ensure that the sample average realized variance equals the sample variance of daily log-returns. For a given threshold κ, we decompose the realized variance into upside and downside realized variances following Barndorff-Nielsen, Kinnebrock, and Shephard (2010):

$$RV_t^U(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t}>\kappa]}, \quad (1)$$

$$RV_t^D(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t}\leq\kappa]}. \quad (2)$$

We add the squared overnight "positive" log-return (exceeding the threshold κ) to the upside realized variance RV_t^U, and the squared overnight "negative" log-return (falling below the threshold κ) to the downside realized variance RV_t^D. Because the daily realized variance sums the upside and the downside realized variances, we apply the same scale to the two components of the realized variance. Specifically, we multiply both components by the ratio of the sample variance of daily log-returns over the sample average of the (pre-scaled) realized variance.

For a given horizon h, we obtain the cumulative realized quantities by summing the one-day realized quantities over h periods:

$$\begin{aligned} RV_{t,h}^U(\kappa) &= \sum_{j=1}^{h} RV_{t+j}^U(\kappa), \\ RV_{t,h}^D(\kappa) &= \sum_{j=1}^{h} RV_{t+j}^D(\kappa), \\ RV_{t,h} &= \sum_{j=1}^{h} RV_{t+j}(\kappa). \end{aligned} \quad (3)$$

By construction, the cumulative realized variance adds up the cumulative realized upside and downside variances:

$$RV_{t,h} \equiv RV_{t,h}^U(\kappa) + RV_{t,h}^D(\kappa). \quad (4)$$

2.2 Disentangling upside from downside variation: A theoretical overview

This section briefly reviews the main theoretical results that allow us to separate daily positive from negative quadratic variation using intraday data. In what follows, the threshold κ is set to 0. We largely rely on Barndorff-Nielsen, Kinnebrock, and Shephard (2010), who assume that the stock price follows a jump-diffusion of the form

$$ds_t = \mu_t dt + \sigma_t dW_t + \Delta s_t,$$

where dW_t is an increment of the standard Brownian motion and $\Delta s_t \equiv s_t - s_{t-}$ refers to the jump component. The instantaneous variance can be defined as $\widetilde{\sigma}_t^2 = \sigma_t^2 + (\Delta s_t)^2$. Under this general assumption on the instantaneous return process, Barndorff-Nielsen, Kinnebrock, and Shephard (2010) use infill asymptotics – asymptotics as the time distance between any two records shrinks toward 0 – to demonstrate that

$$\begin{aligned} RV_{t,h}^U(0) &\overset{p}{\to} \frac{1}{2}\int_t^{t+h} \sigma_\upsilon^2 d\upsilon + \sum_{t\leq\upsilon\leq t+h} (\Delta s_\upsilon)^2\, \mathbb{I}_{[\Delta s_\upsilon > 0]}, \\ RV_{t,h}^D(0) &\overset{p}{\to} \frac{1}{2}\int_t^{t+h} \sigma_\upsilon^2 d\upsilon + \sum_{t\leq\upsilon\leq t+h} (\Delta s_\upsilon)^2\, \mathbb{I}_{[\Delta s_\upsilon \leq 0]}. \end{aligned}$$

Hence, $RV_{t,h}^D(0)$ and $RV_{t,h}^U(0)$ provide a new source of information, which focuses on squared negative and positive jumps, as pointed out by Patton and Sheppard (2015). We present a detailed discussion of these results in an Online Appendix.

2.3 Construction of the variance risk premium components

Next, we characterize the VRP of BTZ through premia accrued to bearing upside and downside variance risks, following these steps:

$$\begin{aligned} VRP_{t,h} &= \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}], \\ &= \left(\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^U(\kappa)]\right) + \left(\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)] - \mathbb{E}_t^{\mathbb{P}}[RV_{t,h}^D(\kappa)]\right), \\ VRP_{t,h} &\equiv VRP_{t,h}^U(\kappa) + VRP_{t,h}^D(\kappa). \end{aligned} \tag{5}$$

Eq. (5) represents the decomposition of the VRP of BTZ into upside and downside variance risk premia – $VRP^U_{t,h}(\kappa)$ and $VRP^D_{t,h}(\kappa)$, respectively – that lies at the heart of our analysis.

2.3.1 Construction of $\mathbb{P}$-expectation

The goal here is to evaluate $\mathbb{E}^{\mathbb{P}}_t[RV^U_{t,h}(\kappa)]$ and $\mathbb{E}^{\mathbb{P}}_t[RV^D_{t,h}(\kappa)]$. To this end, we consider three specifications: Random walk (RW), upside/downside heteroscedastic autoregressive realized variance (U/D-HAR), and multivariate heteroscedastic autoregressive realized variance (M-HAR).

Both U/D-HAR and M-HAR specifications mimic Corsi (2009)'s HAR-RV model. To get genuine expected values for realized measures that are not contaminated by forward bias or the use of contemporaneous data, we perform an out-of-sample forecasting exercise to predict the three realized variances, at different horizons, corresponding to 1, 2, 3, 6, 9, 12, 18, and 24 months ahead. We find that these alternative specifications provide qualitatively similar results, probably due to persistence in volatility. Hence, for simplicity and to save space, we only report the results based on the random walk model.

The random walk model is specified as

$$\mathbb{E}^{\mathbb{P}}_t[RV^{U/D}_{t,h}(\kappa)] = RV^{U/D}_{t-h,h}(\kappa),$$

where U/D stands for "U or D". This is the model used in BTZ.[3]

2.3.2 Construction of $\mathbb{Q}$-expectation

To build the risk-neutral expectation of $RV_{t,h}$, we follow the methodology of Andersen and Bondarenko (2007):

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}_t[RV^U_{t,h}(\kappa)] &\approx \mathbb{E}^{\mathbb{Q}}_t\Big[\int_t^{t+h} \widetilde{\sigma}^2_\upsilon \mathbb{I}_{[\ln(F_\upsilon/F_t)>\kappa_F]} d\upsilon\Big], \\ &= \mathbb{E}^{\mathbb{Q}}_t\Big[\int_t^{t+h} \widetilde{\sigma}^2_\upsilon \mathbb{I}_{[F_\upsilon>F_t\exp(\kappa_F)]} d\upsilon\Big],\end{aligned}$$

[3] Specifications for U/D-HAR and M-HAR models are available in an Online Appendix.

where κ_F is a threshold used to compute risk-neutral expectations of semi-variances.[4] Thus,

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)] &\approx 2\int_{F_t \exp(\kappa_F)}^{\infty} \frac{M_0(\underline{S})}{\underline{S}^2} d\underline{S}, \\ M_0(\underline{S}) &= \min(P_t(\underline{S}), C_t(\underline{S})), \end{aligned} \tag{6}$$

where, $P_t(\underline{S}), C_t(\underline{S})$, and $\underline{S}$ are prices of European put and call options (with maturity h), and the strike price of the underlying asset, respectively. F_t is the price of a future contract at time t, defined as $F_t = S_t \exp(r_t^f h)$. Similarly for $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]$, we get

$$\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)] \approx 2\int_0^{F_t \exp(\kappa_F)} \frac{M_0(\underline{S})}{\underline{S}^2} d\underline{S}. \tag{7}$$

We simplify our notation by renaming $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)]$ and $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]$ as

$$IV_{t,h}^U = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)], \tag{8}$$

$$IV_{t,h}^D = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]. \tag{9}$$

We refer to $IV_{t,h}^{U/D}$ as the "risk-neutral semi-variance" or "implied semi-variance" of returns. These quantities are conditioned on the threshold value κ, which we suppress to simplify notation. As evident in this section, our measures of realized and implied volatility are model-free.

2.4 Construction of the skewness risk premium

The difference between realized upside and downside variance can be perceived as a measure of (realized) skewness, see Feunou, Jahan-Parvar, and Tédongap (2013, 2016). To build this measure of skewness, denoted as $RSV_{t,h}$, we simply subtract downside variance from upside semi-variance:

$$RSV_{t,h}(\kappa) = RV_{t,h}^U(\kappa) - RV_{t,h}^D(\kappa). \tag{10}$$

Thus, if $RSV_{t,h}(\kappa) < 0$ the distribution is left-skewed and when $RSV_{t,h}(\kappa) > 0$ it is right-skewed.

A theoretical justification for using $RSV_{t,h}$ as a measure of skewness can be found in Barndorff-

[4] Note that κ_F should be set to $(\kappa - r_t^f)h$ to get consistent thresholds when computing realized and option-implied quantities.

Nielsen, Kinnebrock, and Shephard (2010), and Feunou, Jahan-Parvar, and Tédongap (2016). A more detailed discussion is available in an Online Appendix.

We construct a measure of skewness risk premium (SRP), which closely resembles the variance risk premium. The SRP is defined as the difference between risk-neutral and objective expectations of the realized skewness. It can be shown that this measure of the skewness risk premium is the spread between the upside and downside components of the variance risk premium:

$$\begin{aligned} SRP_{t,h} &= \mathbb{E}_t^{\mathbb{Q}}[RSV_{t,h}] - \mathbb{E}_t^{\mathbb{P}}[RSV_{t,h}], \\ SRP_{t,h} &= VRP_{t,h}^{U}(\kappa) - VRP_{t,h}^{D}(\kappa). \end{aligned} \tag{11}$$

If $RSV_{t,h} < 0$, we view $SRP_{t,h}$ as a skewness premium – the compensation for an agent who bears downside risk. Alternatively, if $RSV_{t,h} > 0$, we view $SRP_{t,h}$ as a skewness discount – the amount that the agent is willing to pay to secure a positive return on an investment. This measure of the skewness risk premium is nonparametric and model-free.

3 Data

In this study, we adapt BTZ's methodology and use modified measures introduced in Section 2.3. We thus need suitable data to construct excess returns, realized semi-variances ($RV^{U/D}$), and risk-neutral semi-variances ($IV^{U/D}$). Throughout the study, we set $\kappa = 0$.

3.1 Excess returns

We first compute the excess returns by subtracting 3-month treasury bill rates from log-differences in the S&P 500 composite index, sampled at the end of each month. Since our study requires reliable high-frequency data and option-implied volatilities, our sample runs from September 1996 to March 2016. Panel A of Table 1 reports the descriptive statistics of monthly S&P 500 excess return series.

3.2 High-frequency data and realized variance components

We then use intraday S&P 500 data, downloaded from the Institute of Financial Markets, to construct the daily $RV^{U/D}$s series. We sum the five-minute squared negative returns for the downside realized variance (RV^D) and the five-minute squared positive returns for the upside realized variance (RV^U). We next add the daily squared overnight negative returns to the downside semi-variance, and the daily squared overnight positive returns to the upside realized variance. The overnight returns are computed for 4:00 pm to 9:30 am. The total realized variance is obtained by adding the downside and the upside realized variances.[5] To construct physical expectations of volatility measures, we use a random walk model to forecast the three realized variances at horizons ranging between 1 and 24 months ahead.

3.3 Options data and risk-neutral variances

We extract risk-neutral quantities from daily data of European-style put and call options on the S&P 500 index, available through OptionMetrics Ivy database. To obtain consistent risk-neutral moments, we preprocess the data by applying the same filters as in Chang, Christoffersen, and Jacobs (2013). Risk-neutral upside and downside variances ($IV^{U/D}$) are constructed using the model-free *corridor* implied volatility methodology discussed in Andersen, Bondarenko, and Gonzalez-Perez (2015), Andersen and Bondarenko (2007), and Carr and Madan (1999), among others.[6]

Panel B of Table 1 shows that risk-neutral volatility measures are persistent – $AR(1)$ parameters are all above 0.60 – and demonstrate significant skewness and excess kurtosis. It is also clear that the main factor behind volatility behavior is the downside variance. The contribution of upside volatility to risk-neutral volatility is considerably less than that of downside volatility.

4 Empirical Results

In this section, we provide economic intuition and empirical support for our proposed decomposition of the variance risk premium. First, we describe the intuitively expected behavior of the components of the variance risk premium, as well as some salient features of the size and variability of these components. Since our approach is nonparametric, these facts stand as guidelines for realistic

[5] For the three series, we use a multiplicative scaling of the average total realized variance series to match the unconditional variance of S&P 500 returns. Hansen and Lunde (2006) discuss various approaches to adjusting open-to-close RVs.

[6] Our data set contains a large set of option contracts. Moreover, the sample features comparable numbers of out-of-the-money (OTM) put and call contracts (especially in longer-horizon maturities from 18 to 24 months) that enable a precise computation of risk-neutral semi-variances. A detailed description of options data is provided in an Online Appendix.

models (reduced-form and general equilibrium). Second, we provide an extensive investigation of the predictability of the equity premium, based on the variance premium and its components. We empirically demonstrate the contribution of (relative) downside risk premium and characterize the sources of VRP predictability documented by BTZ. Third, we provide a comprehensive robustness study.[7]

4.1 Description of the variance risk premium components

We view the VRP as the premium that a market participant is willing to pay to hedge against variations in future realized volatilities. It is expected to be positive, as rationalized within the general equilibrium model of BTZ, where it is shown to be in general positive and proportional to the volatility-of-volatility. We confirm these findings by reporting the summary statistics for the VRP in Table 1. We also plot the time series of the VRP, its components, and the SRP in Figure 1. We present measures based on random walk and univariate U/D-HAR forecasts of realized volatility and its components under the physical measure. Construction of quantities based on multivariate HAR (M-HAR) are virtually identical to those obtained from univariate HAR.[8] To save space, and since the results obtained from the random walk or HAR methods are quite similar, we only report findings based on random walk forecasts of realized volatility. The series plotted in Figure 1 demonstrate that while HAR-based quantities are more volatile than time series based on the random walk, the difference is mainly due to the magnitude of fluctuations, but not in the fluctuations themselves. This observation may explain the similarities in empirical findings. As expected, from 1996 to 2015, we can see that the variance risk premium is positive most of the time and remains high in uncertain episodes.

We argued in Section 5 that we intuitively expect negative-valued VRP^U and positive-valued VRP^D. Table 1 clearly illustrates these intuitions, as the average VRP^U is about -2.60%. Moreover, Figure 1 confirms that VRP^U is usually negative through our sample period. The same table reports average VRP^D of roughly 5.21%, and in Figure 1, we observe that VRP^D is usually positive. In Section 5, we show that under mild assumptions these expectations about VRP components are supported by our theoretical model.

[7]Additional robustness results are available in an Online Appendix.

[8]U/D-HAR and M-HAR forecasts of realized volatility and its components are based on the methodology of Corsi (2009).

Building on the same intuition and methodology, we show that the sign of the SRP, which stems from the expected behavior of the two components of the VRP, is negative. Indeed, Table 1 reports that the average skewness risk premium is -7.8%. In Figure 1, we clearly observe that SRP is generally negative-valued.

Table 1 also reveals highly persistent, negatively skewed, and fat-tailed empirical distributions for (downside/upside) variance and skewness risk premia. Nonetheless, upside variance and skewness risk premia appear more left-skewed and leptokurtic as compared to (total) variance and downside variance risk premia.

4.2 Predictability of the equity premium

BTZ derive a theoretical model where the VRP emerges as the main driver of time variation in the equity premium. They show both theoretically and empirically that a higher VRP predicts higher future excess returns. Intuitively, the variance risk premium proxies the premium associated with the volatility-of-volatility, which not only reflects how future random returns vary but also assesses fluctuations in the tail thickness of future returns distribution.

Because the VRP sums downside and upside variance risk premia, BTZ's framework entails imposing the same coefficient on both (upside and downside) components of the VRP when they are jointly included in a predictive regression of excess returns. However, such a constraint seems very restrictive, given the asymmetric views of investors on good uncertainty – proneness to upward variability – versus bad uncertainty – aversion to downward variability. Sections 5 and 4.1 document that both VRP^D and VRP^U have intrinsically different features.

It is important to point out that by highlighting the disparities between upside and downside variance risk premia, and similar to Bollerslev and Todorov (2011) and Bollerslev, Todorov, and Xu (2015), our study intends to push the findings of BTZ further. BTZ's study is undertaken to rationalize the importance of the variance risk premium in explaining the dynamics of the equity premium. Our goal is to build on BTZ's framework, showing that introducing asymmetry in the VRP analysis provides additional flexibility to the trade-off between return and second-moment risk premia. Ultimately, our approach is intended to strengthen the concept behind the variance risk premium of BTZ.

Our results are based on a simple linear regression of k-step-ahead cumulative S&P 500 excess returns on values of a set of predictors that include the VRP, VRP^U, VRP^D, and SRP. Following the results of Ang and Bekaert (2007), reported Student's t-statistics are based on heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, as advocated by Hodrick (1992). The model used for our analysis is simply

$$r^e_{t\to t+k} = \beta_0 + \beta_1 x_t(h) + \epsilon_{t\to t+k}, \tag{12}$$

where $r^e_{t\to t+k}$ is the cumulative excess returns between time t and $t+k$, $x_t(h)$ is one of the predictors discussed in Sections 2.3 and 2.4 at time t, h is the construction horizon of $x_t(h)$, and $\epsilon_{t\to t+k}$ is a zero-mean error term. We focus our discussion on the significance of the estimated slope coefficients (β_1s), determined by the robust Student-t statistics. We report the predictive ability of regressions, measured by the corresponding adjusted R^2s. For highly persistent predictor variables, the R^2s for the overlapping multi-period return regressions must be interpreted with caution, as noted by BTZ and Jacquier and Okou (2014), among others.

We decompose the contribution of our predictors to show that (1) predictability results documented by BTZ are driven by the downside variance risk premium, (2) predictability results are mainly driven by risk-neutral expectations – thus, risk-neutral measures contribute more than realized measures – and (3) the contribution of the skewness risk premium increases as a function of both the predictability horizon (k) and the aggregation or maturity horizon (h).

Our empirical findings, presented in Tables 2 to 5, support all three claims. In Panel A of Table 2, we show that the two main regularities uncovered by BTZ, the hump-shaped increase in robust Student-t statistics and adjusted R^2s reaching their maximum at $k = 3$ (one quarter ahead), are present in the data. These effects, however, weaken as the aggregation horizon (h) increases from one month to three months or more; the predictability pattern weakens and then largely disappears for $h > 6$.

Panel B of Table 2 reports the predictability results based on using VRP^D as the predictor. We observe the hump-shaped pattern for Student's t-statistics and the adjusted R^2s reaching their maximum between $k = 3$ or $k = 6$ months. Moreover, these results are more robust to the aggregation horizon of the predictor. We notice that, in contrast to the VRP results – where predictability is only present for monthly or quarterly constructed risk premia – the VRP^D results are largely robust to aggregation horizons; the slope parameters are statistically different from zero even for annually constructed downside variance risk premia ($h = 12$). Moreover, the VRP^D results yield higher adjusted R^2s compared with the VRP regressions at similar prediction horizons, an observation that we interpret as the superior ability of the VRP^D to explain the variation in aggregate excess returns. Last, but not least, we notice a gradual shift in prediction results from the familiar one-quarter-ahead peak of predictability documented by BTZ to 9-to-12-months-ahead peaks, once we increase the aggregation horizon h. Based on these results, we infer that the VRP^D is the likely candidate to explain the predictive power of VRP, documented by BTZ.

Our results for predictability based on the VRP^U, reported in Panel C of Table 2 are weak. The hump-shaped pattern in both robust Student's t-statistics and in adjusted R^2s, while present, is significantly weaker than the results reported by BTZ. Once we increase the aggregation horizon, h, these results are lost. We conclude that bearing upside variance risk does not appear to be an important contributor to the equity premium and, hence, is not a good predictor of this quantity. In addition, we interpret these findings as a low contribution of the VRP^U to overall VRP.

We observe a new set of interesting regularities when we use the SRP as our predictor. These results are reported in Panel D of Table 2 . It is immediately clear that the SRP displays a stronger predictive power at longer horizons than the VRP. For monthly excess returns, the SRP slope coefficient is statistically different from zero at prediction horizons of 6 months ahead or longer. At $k = 6$, the adjusted R^2 of the SRP is comparable in size with that of the VRP (2.30% against 3.65%, respectively) and is strictly greater thereafter. At $k = 6$, the adjusted R^2 for the monthly excess return regression based on the SRP is smaller than that of the VRP^D. However, their sizes are comparable at $k = 9$ and $k = 12$ months ahead. Both trends strengthen as we consider higher aggregation levels for excess returns. At the semi-annual construction level ($h = 6$), the SRP already has more predictive power than both the VRP and VRP^D at a quarter-ahead prediction horizon. The increase in adjusted R^2s of the SRP is not monotonic in the construction horizon level. We can detect a maximum at a roughly three-quarters-ahead prediction window for semi-annual and annually constructed SRP. This observation implies that the SRP is the intermediate link between one-quarter-ahead predictability using the VRP uncovered by BTZ and the long-term predictors such as the price-dividend ratio, dividend yield, or consumption-wealth ratio of Lettau and Ludvigson (2001). Given the generally unfavorable findings of Goyal and Welch (2008) regarding long-term predictors of equity premium, our findings regarding the predictive power of the SRP are particularly encouraging.

At this point, it is natural to inquire about including both VRP components in a predictive regression. We present the empirical evidence from this estimation in Panel A of Table 5. After inclusion of the VRP^U and VRP^D in the same regression, the statistical significance of the VRP^U's slope parameters are broadly lost. We also notice a sign change in Student's t-statistics associated with the estimated slope parameters of the VRP^U and VRP^D. This observation is not surprising. As we show in our equilibrium model, and also intuitively, risk-averse investors like variability in positive outcomes of returns but dislike it in negative outcomes. Hence, in a joint regression, we expect the coefficient of VRP^D to be positive and that of VRP^U to be negative. This observation, as documented in Feunou, Jahan-Parvar, and Tédongap (2013), lends additional credibility to the role of the SRP as a predictor of aggregated excess returns.[9]

We claim that the patterns discussed earlier, and, hence, the predictive power of the VRP, VRP^D, and SRP are rooted in expectations. That is, the driving force behind our results, as well as those of BTZ, are expected risk-neutral measures of the volatility components. To show the empirical findings supporting our claim, we run predictive regressions using equation (12). Instead of using the "premia" employed so far, we use realized and risk-neutral measures of variances, up- and downside variances, and skewness for x_t, based on our discussions in Section 2, respectively.

Our empirical findings using risk-neutral volatility measures are available in Table 3. In Panel A, we report the results of running a predictive regression when the predictor is the risk-neutral variance obtained from direct application of the Andersen and Bondarenko (2007) method. It is clear that the estimated slope parameters are statistically different from zero for $k \geq 3$ at most construction horizons, h. The reported adjusted R^2s also imply that the predictive regressions have explanatory power for aggregate excess return variations at $k \geq 3$. The same patterns are discernible for risk-neutral downside and upside variances (Panels B and C) and risk-neutral skewness (Panel

[9]Briefly, based on arguments similar to those advanced by Feunou, Jahan-Parvar, and Tédongap (2013), we expect estimated parameters of the VRP^U and VRP^D to have opposite signs, and be statistically "close." As such, they imply that the SRP is the predictor we should have included instead of these VRP components.

D). Adjusted R^2s reported are lower than those reported in Table 2, and these measures of variation yield statistically significant slope parameters at longer prediction horizons than what we observe for the VRP and its components. Taken together, these observations imply that using the premium (rather than the risk-neutral variation) yields better predictions.

However, in comparison with realized (physical) variation measures, risk-neutral measures yield better results. The analysis using realized variation measures is available in Table 4. It is obvious that, by themselves, the realized measures do not yield reasonable predictability, an observation corroborated by the empirical findings of Bekaert, Engstrom, and Ermolov (2015). The majority of the estimated slope parameters are statistically indistinguishable from zero, and the adjusted R^2s are low. Inclusion of both risk-neutral or realized variance components does not change our findings dramatically, as demonstrated in Panels B and C of Table 5.

We observe in Panel D of Table 4 and in Panel C of Table 5 statistical significance and notable adjusted R^2s for realized skewness in long prediction horizons ($k \geq 6$) and for construction horizons ($h \geq 6$). By itself (as opposed to the SRP studied earlier), the realized skewness lacks predictive power in low construction or prediction horizons. Based on our results presented in Table 2, we argue that the SRP (and not the realized skewness) is a more suitable predictor, as it overcomes these two shortcomings.

Given the weak performance of realized measures, it is easy to conclude that realized variation plays a secondary role to risk-neutral variation measures in driving the predictability results documented by BTZ or in this study. However, we need both elements in the construction of the variance or skewness risk premia, since realized or risk-neutral measures individually possess inferior prediction power.

4.3 Robustness

We perform extensive robustness exercises to document the prediction power of the VRP^D and SRP for aggregate excess returns in the presence of traditional predictor variables. The goal is to highlight the contribution of our proposed variables in a wider empirical context. Simply put, we observe that the predictive power does not disappear when we include other pricing variables, implying that the VRP^D and SRP are not simply proxies for other well-known pricing ratios.

Following BTZ and Feunou et al. (2014), among many others, we include equity pricing measures such as the log price-dividend ratio ($\log(p_t/d_t)$), lagged log price-dividend ratio ($\log(p_{t-1}/d_t)$), and log price-earnings ratio ($\log(p_t/e_t)$); yield and spread measures such as term spread (tms_t), the difference between 10-year U.S. Treasury Bond yields and 3-month U.S. Treasury Bill yields; default spread (dfs_t), defined as the difference between BBB and AAA corporate bond yields; CPI inflation ($infl_t$); and, finally, Kelly and Pruitt (2013) partial least-squares-based, cross-sectional in-sample and out-of-sample predictors ($kpis_t$ and $kpos_t$, respectively).

We consider two periods for our analysis: our full sample – September 1996 to December 2010 – and a pre-Great Recession sample, September 1996 to December 2007. The latter ends at the same point in time as the BTZ sample. We report our empirical findings in Tables 6 to 9. These results are based on semi-annually aggregated excess returns and estimated for the one-month-ahead prediction horizon.[10] In this robustness study, we scale the cumulative excess returns; we use $r^e_{t\to t+6}/6$ as the predicted value and regress it on a one-month lagged predictive variable.

Full-sample simple predictive regression results are available in Table 6. Among VRP components, only the downside variance risk premium ($dvrp_t$) and the skewness risk premium (srp_t) have slope parameters that are statistically different from zero and have adjusted R^2s comparable in magnitude with other pricing variables. Once we use $dvrp_t$ along with other pricing variables, we observe the following regularities in Table 7, which reports the joint multivariate regression results. First, the estimated slope parameter for $dvrp_t$ is statistically different from zero in all cases, except when we include srp_t. This result is not surprising, since srp_t and $dvrp_t$ are linearly dependent. Second, these regressions yield adjusted R^2s that range between 3.10% (for $dvrp_t$ and tms_t, in line with findings of BTZ that report weak predictability for tms_t) to 25.71% (for $dvrp_t$ and $infl_t$).[11] The downside variance risk premium in conjunction with the variance risk premium or upside variance risk premium remains statistically significant and yields adjusted R^2s that are in the 7% neighborhood.

[10] A complete set of robustness checks, including monthly, quarterly, and annually aggregated excess returns results, are available in an online Appendix.

[11] The dynamics of inflation during the Great Recession period mimic the behavior of our variance risk premia. Gilchrist et al. (2014) meticulously study the behavior of this variable in the 2007 – 2009 period. According to their study, both full and matched PPI inflation in their model display an aggregate drop in 2008 and 2009, while the reaction of financially sound and weak firms are asymmetric, with the former lowering prices and the latter raising prices in this period. Thus, the predictive power of this variable, given the inherent asymmetric responses, is not surprising.

We obtain adjusted R^2s that are decidedly lower than those reported by BTZ for quarterly and annually aggregated multivariate regressions. These differences are driven by the inclusion of the Great Recession period data in our full sample. To illustrate this point, we repeat our estimation with the data set ending in December 2007. Simple predictive regression results based on this data are available in Table 8. We immediately observe that the exclusion of the Great Recession period data improves even the univariate predictive regression adjusted R^2s across the board. The estimated slope parameters are also closer to BTZ estimates and are generally statistically significant.

In Table 9, we report multivariate regression results, based on 1996 – 2007 data. We notice that once $dvrp_t$ is included in the regression model, the variance risk premium, upside variance risk premium, and skewness risk premium are no longer statistically significant. Other pricing variables – except for term spread, default spread, and inflation – yield slope parameters that are statistically significant. Thus, inflation seems to lack prediction power in this sub-sample. We do not observe statistically insignificant slope parameters for the downside variance risk premium except when we include vrp_t. Across the board, adjusted R^2s are high in this sub-sample.

5 A simple equilibrium model

In this section, we present an equilibrium consumption-based asset pricing model that supports the proposed decomposition of the VRP in terms of upside and downside components. We estimate this model, using a maximum likelihood procedure. We view this exercise as a theoretical motivation for our empirical findings. Our main objective is to highlight the roles that upside and downside variances play in pricing a risky asset in an otherwise standard asset pricing model. In particular, we show that the model, under standard and mild assumptions, (1) yields closed-form solutions for VRP components and SRP that align well with empirical regularities, (2) yields VRP components and SRP that have the expected signs observed in our empirical findings, and (3) fitting the model to data yields empirical results that align well with model predictions, empirical regularities discussed in Section 4, and with BTZ findings. To save space, we only report the main results. The appendix reports step-by-step derivations of the theoretical findings and the estimation procedure.

5.1 Preferences

We consider an endowment economy in discrete time. The representative agent's preferences over the future consumption stream are characterized by Kreps and Porteus (1978) intertemporal preferences, as formulated by Epstein and Zin (1989) and Weil (1989)

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{13}$$

where C_t is the consumption bundle at time t, δ is the subjective discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of intertemporal substitution (IES).[12] Parameter θ is defined as $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. If $\theta = 1$, then $\gamma = 1/\psi$ and Kreps and Porteus preferences collapse to expected power utility, which implies an agent who is indifferent to the timing of the resolution of the uncertainty of the consumption path. With $\gamma > 1/\psi$, the agent prefers early resolution of uncertainty. For $\gamma < 1/\psi$, the agent prefers late resolution of uncertainty. Epstein and Zin (1989) show that the logarithm of the stochastic discount factor (SDF) implied by these preferences is given by

$$\ln M_{t+1} = m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \tag{14}$$

where $\Delta c_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right)$ is the log growth rate of aggregate consumption, and $r_{c,t}$ is the log return of the asset that delivers aggregate consumption as dividends. This asset represents the returns on a wealth portfolio. The Euler equation states that

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1, \tag{15}$$

where $r_{c,t}$ represents the log returns for the consumption-generating asset ($r_{c,t}$). The risk-free rate, which represents the returns on an asset that delivers a unit of consumption in the next period with certainty, is defined as

$$r_t^f = \ln\left[\frac{1}{\mathbb{E}_t(M_{t+1})}\right]. \tag{16}$$

[12]Our theoretical findings do not depend on our choice for preferences. We use EZ in order to make our results comparable with BTZ, Segal, Shaliastovich, and Yaron (2015) or other similar papers. We have not attempted this exercise, but one can follow Bekaert and Engstrom (2015) and use habit-based preferences to obtain very similar theoretical results. The only additional ingredient needed in Bekaert and Engstrom "BE-GE" set up is time-varying dynamics for both volatility-of-volatility components.

5.2 Consumption dynamics under the physical measure

Our specification of consumption dynamics incorporates elements from Bansal and Yaron (2004), BTZ, Feunou, Jahan-Parvar, and Tédongap (2013), Segal, Shaliastovich, and Yaron (2015), Bekaert and Engstrom (2015), and Kilic and Shaliastovich (2015).

Fundamentally, we follow Bansal and Yaron (2004) in assuming that consumption growth has a predictable component. We differ from Bansal and Yaron in assuming that the predictable component is proportional to consumption growth's upside and downside volatility components. Thus, we are closer to Segal, Shaliastovich, and Yaron (2015).[13] As a result, we have

$$\Delta c_{t+1} = \mu_0 + \mu_1 V_{u,t} + \mu_2 V_{d,t} + \sigma_c \left(\varepsilon_{u,t+1} - \varepsilon_{d,t+1}\right), \tag{17}$$

where $\mu_1, \mu_2 \in \mathbb{R}$, $\varepsilon_{u,t+1}$ and $\varepsilon_{d,t+1}$ are two mean-zero shocks that affect both the realized and expected consumption growth.[14] $\varepsilon_{u,t+1}$ represents upside shocks to consumption growth, and $\varepsilon_{d,t+1}$ stands for downside shocks. Following Bekaert and Engstrom (2015) and Segal, Shaliastovich, and Yaron (2015), we assume that these shocks have a demeaned Gamma distribution and model them as

$$\varepsilon_{i,t+1} = \tilde{\varepsilon}_{i,t+1} - V_{i,t},\ i = \{u, d\}, \tag{18}$$

where $\tilde{\varepsilon}_{i,t+1} \sim \Gamma(V_{i,t}, 1)$.[15] These distributional assumptions imply that volatilities of upside and downside shocks are time-varying and driven by shape parameters $V_{u,t}$ and $V_{d,t}$. In particular, we have that

$$Var_t[\varepsilon_{i,t+1}] = V_{i,t},\ i = \{u, d\}. \tag{19}$$

Naturally, the total conditional variance of consumption growth when $\varepsilon_{u,t+1}$ and $\varepsilon_{d,t+1}$ are conditionally independent is simply $\sigma_c^2 \left(V_{u,t} + V_{d,t}\right)$.

[13] Feunou, Jahan-Parvar, and Tédongap (2013), Bekaert and Engstrom (2015) and Kilic and Shaliastovich (2015) consider variations of this assumption in their studies.

[14] This assumption is for the sake of brevity. Violating this assumption adds to algebraic complexity but does not affect our analytical findings.

[15] Assuming demeaned Gamma distributions is for the sake of tractability. One can use a number of alternative distributions with positive support and fat tails. For example, one may choose from compound Poisson, χ^2, inverse Gaussain, or Lévy distributions.

Our specification of the consumption growth distribution is motivated by the theoretical results discussed in Section 2.4 and in the Appendix. In the Appendix, we show that the distribution of the shock to return shares similar properties as the difference of demeaned Gamma in equation (17): (i) the variance is the sum of the variances of upside and downside shocks, and (ii) the skewness is (up to a scaling factor) the difference between the variances of upside and downside shocks.

The sign and size of μ_1 and μ_2 matter in this context. With $\mu_1 = \mu_2$, we have a stochastic volatility component in the conditional mean of the consumption growth process, similar to the classic GARCH-in-Mean structure for modeling risk-return trade-off in equity returns. With both slope parameters equal to zero, the model yields the BTZ unpredictable consumption growth.[16] If $|\mu_1| = |\mu_2|$, with $\mu_1 > 0$ and $\mu_2 < 0$, we have Skewness-in-Mean, similar in spirit to the Feunou, Jahan-Parvar, and Tédongap (2013) formulation for equity returns. With $\mu_1 \neq \mu_2$, we have free parameters that have an impact on loadings of risk factors on risky asset returns and the stochastic discount factor. Intuitively, we expect $\mu_1 > 0$: A rise in upside volatility at time t implies higher consumption growth at time $t+1$, all else being equal. By the same logic, we intuitively expect a negative-valued μ_2, implying an expected fall in consumption growth following an uptick in downside volatility – following bad economic outcomes, households curb their consumption.

We observe that

$$\ln \mathbb{E}_t \exp\left(\nu \varepsilon_{i,t+1}\right) = f(\nu) V_{i,t}, \tag{20}$$

where $f(\nu) = -(\ln(1-\nu) + \nu)$. Both Bekaert, Engstrom, and Ermolov (2015) and Segal, Shaliastovich, and Yaron (2015) use this compact functional form for the Gamma distribution cumulant. It simply follows that $f(\nu) > 0, f''(\nu) > 0$, and $f(\nu) > f(-\nu)$ for all $\nu > 0$.

We assume that $V_{i,t}$ follows a time-varying, square root process with time-varying volatility-of-

[16] A consumption-based asset pricing model with a representative agent endowed with Epstein and Zin (1989) preferences and an unpredictable consumption growth process does not support the existence of distinct upside and downside variance risk premia with the expected signs. In particular, we have found that such a setting always yields a positive upside variance risk premium.

volatility, similar to the specification of the volatility process in BTZ:

$$V_{u,t+1} = \alpha_u + \beta_u V_{u,t} + \sqrt{q_{u,t}} z_{t+1}^u, \tag{21}$$

$$q_{u,t+1} = \gamma_{u,0} + \gamma_{u,1} q_{u,t} + \varphi_u \sqrt{q_{u,t}} z_{t+1}^1, \tag{22}$$

$$V_{d,t+1} = \alpha_d + \beta_d V_{d,t} + \sqrt{q_{d,t}} z_{t+1}^d, \tag{23}$$

$$q_{d,t+1} = \gamma_{d,0} + \gamma_{d,1} q_{d,t} + \varphi_d \sqrt{q_{d,t}} z_{t+1}^2, \tag{24}$$

where z_t^i are standard normal innovations, and $i = \{u, d, 1, 2\}$. The parameters must satisfy the following restrictions: $\alpha_u > 0, \alpha_d > 0, \gamma_{u,0} > 0, \gamma_{d,0} > 0, |\beta_u| < 1, |\beta_d| < 1, |\gamma_{u,1}| < 1, |\gamma_{d,1}| < 1, \varphi_u > 0, \varphi_d > 0$. In addition, we assume that $\{z_t^u\}, \{z_t^d\}, \{z_t^1\}$, and $\{z_t^2\}$ are *i.i.d.* $\sim N(0,1)$ and jointly independent from $\{\varepsilon_{u,t}\}$ and $\{\varepsilon_{d,t}\}$.

The assumptions above yield time-varying uncertainty and asymmetry in consumption growth. Through volatility-of-volatility processes $q_{u,t}$ and $q_{d,t}$, the setup induces additional temporal variation in consumption growth. Temporal variation in the volatility-of-volatility process is necessary for generating a sizeable variance risk premium. Asymmetry is needed to generate upside and downside variance risk premia.

We solve the model following the methodology proposed by Bansal and Yaron (2004), BTZ, and many others. We consider that the logarithm of the wealth-consumption ratio w_t or the price-consumption ratio ($pc_t = \ln\left(\frac{P_t}{C_t}\right)$) for the asset that pays the consumption endowment $\{C_{t+i}\}_{i=1}^{\infty}$ is affine with respect to state variables $V_{i,t}$ and $q_{i,t}$.

We then posit that the consumption-generating returns are approximately linear with respect to the log price-consumption ratio, as popularized by Campbell and Shiller (1988):

$$r_{c,t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + \Delta c_{t+1},$$

$$w_t = A_0 + A_1 V_{u,t} + A_2 V_{d,t} + A_3 q_{u,t} + A_4 q_{d,t},$$

where κ_0 and κ_1 are log-linearization coefficients, and A_0, A_1, A_2, A_3, and A_4 are factor-loading coefficients to be determined. We solve for the consumption-generating asset returns, $r_{c,t}$, using the Euler equation (15). Following standard arguments, we find the equilibrium values of coefficients

A_0 to A_4:

$$A_1 = -\frac{f\Big[\sigma_c(1-\gamma)\Big] + (1-\gamma)\mu_1}{\theta(\kappa_1\beta_u - 1)}, \tag{25}$$

$$A_2 = -\frac{f\Big[-\sigma_c(1-\gamma)\Big] + (1-\gamma)\mu_2}{\theta(\kappa_1\beta_d - 1)}, \tag{26}$$

$$A_3 = \frac{(1-\kappa_1\gamma_{u,1}) - \sqrt{(1-\kappa_1\gamma_{u,1})^2 - \theta^2\varphi_u^2\kappa_1^4 A_1^2}}{\theta\kappa_1^2\varphi_u^2}, \tag{27}$$

$$A_4 = \frac{(1-\kappa_1\gamma_{d,1}) - \sqrt{(1-\kappa_1\gamma_{d,1})^2 - \theta^2\varphi_d^2\kappa_1^4 A_2^2}}{\theta\kappa_1^2\varphi_d^2}, \tag{28}$$

$$A_0 = \frac{\ln\delta + \left(1-\frac{1}{\psi}\right)\mu_0 + \kappa_0 + \kappa_1\left(\alpha_u A_1 + \alpha_d A_2 + \gamma_{u,0}A_3 + \gamma_{d,0}A_4\right)}{1-\kappa_1}. \tag{29}$$

It is easy to see that while A_3 and A_4 are negative-valued, the signs of A_1 and A_2 depend on the signs and sizes of μ_1 and μ_2. We report the conditions that ensure $A_1 > 0$ and $A_2 < 0$ after deriving the dynamic of the model under the risk-neutral measure.

5.3 Risk-neutral dynamics and the premia

Combining the historical dynamic and the stochastic discount factor imply the risk-neutral dynamic and closed-form expression for different risk premia (See the external appendix for details on the mathematical derivations). Starting with the equity risk premium, we have

$$\begin{aligned} ERP_t &\equiv \mathbb{E}_t\left[r_{c,t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[r_{c,t+1}\right] \\ &= \frac{\gamma\sigma_c^2}{1+\gamma\sigma_c}V_{u,t} + \frac{\gamma\sigma_c^2}{1-\gamma\sigma_c}V_{d,t} \\ &\quad +(1-\theta)\kappa_1^2\left(A_1^2 + A_3^2\varphi_u^2\right)q_{u,t} + (1-\theta)\kappa_1^2\left(A_2^2 + A_4^2\varphi_d^2\right)q_{d,t}. \end{aligned} \tag{30}$$

This expression for the equity risk premium shows that our model implies unequal loadings for upside and downside volatility factors. The slope coefficients for volatility-of-volatility factors are also, in general, unequal. To derive the upside and downside variance risk premia, we need to decompose the total variance of $r_{c,t+1}$. The total conditional variance ($\sigma_{r,t}^2 \equiv Var_t\left[r_{c,t+1}\right]$) is given by

$$\sigma_{r,t}^2 = \sigma_c^2 V_{u,t} + \sigma_c^2 V_{d,t} + \kappa_1^2\left(A_1^2 + A_3^2\varphi_u^2\right)q_{u,t} + \kappa_1^2\left(A_2^2 + A_4^2\varphi_d^2\right)q_{d,t}.$$

The upside and downside variances are

$$\left(\sigma_{r,t}^{u}\right)^2 = \sigma_c^2 V_{u,t} + \kappa_1^2\left(A_1^2 + A_3^2\varphi_u^2\right) q_{u,t}, \tag{31}$$

$$\left(\sigma_{r,t}^{d}\right)^2 = \sigma_c^2 V_{d,t} + \kappa_1^2\left(A_2^2 + A_4^2\varphi_d^2\right) q_{d,t}. \tag{32}$$

Hence, the upside and downside variance risk premia are given by

$$VRP_t^U \equiv \mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^{u}\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^{u}\right)^2\right] = (\theta-1)\left(\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right) A_3\varphi_u^2\right) q_{u,t},$$

$$VRP_t^D \equiv \mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^{d}\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^{d}\right)^2\right] = (\theta-1)\left(\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right) A_4\varphi_d^2\right) q_{d,t}.$$

As discussed before, we expect $VRP_t^U < 0$ and $VRP_t^D > 0$. It follows that

$$\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right) A_3\varphi_u^2 > 0, \tag{33}$$

$$\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right) A_4\varphi_d^2 < 0. \tag{34}$$

In the external appendix we discuss the necessary and sufficient conditions ensuring that both inequalities (33) and (34) hold. We can express these conditions in a very simple and intuitive way:

- A sufficient condition for $VRP_t^d > 0$ is $\mu_2 \leq 0$,
- A necessary condition for $VRP_t^u < 0$ is $\mu_1 \geq \frac{f(\sigma_c(1-\gamma))}{\gamma-1} \geq 0$.

Our estimation delivers structural parameter estimates that satisfy all the theoretically expected restrictions – inducing $VRP_t^U < 0$ and $VRP_t^D > 0$ –, and are consistent with the facts discussed in depth in the first part of the paper. Since equation (30) implies that the equity risk premium loads positively on both $q_{u,t}$ and $q_{d,t}$, and because $VRP_t^U < 0$ is negatively proportional to $q_{u,t}$ while $VRP_t^D > 0$ is positively proportional to $q_{d,t}$, the equity risk premium loads positively on the downside variance risk premium but negatively on the upside variance risk premium. This feature of the general equilibrium model is also consistent with the empirical regularities documented in the predictability analysis.

5.4 Estimation

To appraise the empirical performance of our general equilibrium, we implement a maximum likelihood estimation procedure. Namely, we maximize the joint likelihood of consumption growth, stock market return, and risk-free rate series. The following steps provide a brief description of the estimation. The shocks to consumption growth and stock return are

$$\begin{aligned}\Delta c_{t+1} - \mathbb{E}_t\left[\Delta c_{t+1}\right] &= \epsilon_{1,t+1}, \\ r_{c,t+1} - \mathbb{E}_t(r_{c,t+1}) &= \epsilon_{1,t+1} + \epsilon_{2,t+1},\end{aligned}$$

where

$$\begin{aligned}\epsilon_{1,t+1} &\equiv \sigma_c(\varepsilon_{u,t+1} - \varepsilon_{d,t+1}), \\ \epsilon_{2,t+1} &\equiv \kappa_1\left[\left(A_1 z^u_{t+1} + \varphi_u A_3 z^1_{t+1}\right)\sqrt{q_{u,t}} + \left(A_2 z^d_{t+1} + \varphi_d A_4 z^2_{t+1}\right)\sqrt{q_{d,t}}\right],\end{aligned}$$

and $\epsilon_{1,t+1}$, $\epsilon_{2,t+1}$ are conditionally independent random variables. Note that

$$\epsilon_{2,t+1} \sim N\left(0, \kappa_1^2\left(A_1^2 + \varphi_u^2 A_3^2\right) q_{u,t} + \kappa_1^2\left(A_2^2 + \varphi_d^2 A_4^2\right) q_{d,t}\right).$$

Hence, the joint density function of consumption and return writes

$$f_{(c,r_c)}\left(\Delta c_{t+1}, r_{c,t+1}\right) = f_{\epsilon_1}\left(\Delta c_{t+1} - \mathbb{E}_t\left[\Delta c_{t+1}\right]\right) f_{\epsilon_2}\left(r_{c,t+1} - \Delta c_{t+1} - \left(\mathbb{E}_t(r_{c,t+1}) - \mathbb{E}_t(r_{c,t+1})\right)\right),$$

where f_{ϵ_1} and f_{ϵ_2} are the marginal densities of $\epsilon_{1,t+1}$ and $\epsilon_{2,t+1}$, respectively. These marginal densities can be computed according to

$$f_{\epsilon_1}\left(\epsilon_{1,t+1}\right) = \frac{1}{\pi}\int_0^\infty Re\left[\exp\left(-i\nu\epsilon_{1,t+1} + f(i\sigma_c\nu)V_{u,t} + f(-i\sigma_c\nu)V_{d,t}\right)\right] d\nu,$$

and

$$
\begin{aligned}
\ln\left[f_{\epsilon_2}\left(\epsilon_{2,t+1}\right)\right] &= -\frac{1}{2}\ln\left(2\pi\right)-\frac{1}{2}\ln\left(\kappa_1^2\left(A_1^2+\varphi_u^2A_3^2\right)q_{u,t}+\kappa_1^2\left(A_2^2+\varphi_d^2A_4^2\right)q_{d,t}\right)\\
&\quad -\frac{1}{2}\frac{\epsilon_{2,t+1}^2}{\kappa_1^2\left(A_1^2+\varphi_u^2A_3^2\right)q_{u,t}+\kappa_1^2\left(A_2^2+\varphi_d^2A_4^2\right)q_{d,t}}.
\end{aligned}
$$

It follows that the joint log-likelihood of consumption and return is calculated as

$$
\ln L^{(C,R)}=\sum_{t=0}^{T-1}\ln\left(f_{(c,r_c)}\left(\Delta c_{t+1},r_{c,t+1}\right)\right), \tag{35}
$$

where T denotes the sample size. The risk-free rate distribution is based on a Gaussian error likelihood

$$
\ln L^{RF}\propto-\frac{1}{2}\sum_{t=1}^{T}\left\{\ln\left(RFRMSE^2\right)+e_t^2/RFRMSE^2\right\},
$$

where the error term is computed as $e_t=rf_t^{observed}-rf_t^{Model}$, and the corresponding root-mean-square error is given by $RFRMSE\equiv\sqrt{\frac{1}{T}\sum_{t=1}^{T}e_t^2}$. Finally, all the parameters are estimated by maximizing the joint likelihood of consumption growth, stock return, and risk-free rate

$$
\ln L^{(C,R)}+\ln L^{RF}. \tag{36}
$$

To effectively implement our estimation strategy, we need $V_{u,t}$, $V_{d,t}$, $q_{u,t}$ and $q_{d,t}$ that are latent factors in the model. To circumvent this challenge, we assume that observed one-month ahead upside variance risk premium (VRP_t^U), downside variance risk premium (VRP_t^D), conditional upside stock return variance ($\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^U]$), and conditional downside stock return variance ($\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^D]$) are measured without error. This assumption entails that the observed quantities exactly match their theoretical counterparts, thus allowing to infer the latent factors as

$$
\begin{aligned}
q_{u,t} &= -\frac{1}{(1-\theta)\kappa_1\left(\sigma_c^2A_1+\kappa_1^2\left(A_1^2+A_3^2\varphi_u^2\right)A_3\varphi_u^2\right)}VRP_t^U\equiv\varsigma_uVRP_t^U,\\
q_{d,t} &= -\frac{1}{(1-\theta)\kappa_1\left(\sigma_c^2A_2+\kappa_1^2\left(A_2^2+A_4^2\varphi_d^2\right)A_4\varphi_d^2\right)}VRP_t^D\equiv\varsigma_dVRP_t^D,
\end{aligned}
$$

$$
\begin{aligned}
V_{u,t} &= -\frac{\kappa_1^2}{\sigma_c^2}\left(A_1^2 + A_3^2\varphi_u^2\right)\frac{\gamma_{u,0}}{\beta_u} - \frac{\alpha_u}{\beta_u} + \frac{1}{\beta_u\sigma_c^2}\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^U] - \frac{\kappa_1^2}{\sigma_c^2}\left(A_1^2 + A_3^2\varphi_u^2\right)\frac{\gamma_{u,1}}{\beta_u}\varsigma_u VRP_t^U, \\
&\equiv \varpi_u + \vartheta_u\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^U] + \varrho_u VRP_t^U,
\end{aligned}
$$

$$
\begin{aligned}
V_{d,t} &= -\frac{\kappa_1^2}{\sigma_c^2}\left(A_2^2 + A_4^2\varphi_d^2\right)\frac{\gamma_{d,0}}{\beta_d} - \frac{\alpha_d}{\beta_d} + \frac{1}{\beta_d\sigma_c^2}\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^D] - \frac{\kappa_1^2}{\sigma_c^2}\left(A_2^2 + A_4^2\varphi_d^2\right)\frac{\gamma_{d,1}}{\beta_d}\varsigma_d VRP_t^D, \\
&\equiv \varpi_d + \vartheta_d\mathbb{E}_t^{\mathbb{P}}[RV_{t+1}^D] + \varrho_d VRP_t^D.
\end{aligned}
$$

Table (10) reports the structural parameter estimates of the general equilibrium model and their corresponding standard errors. These results clearly show that our general equilibrium model yields accurate parameter estimates that are consistent with the empirical evidence. Specifically, the estimated values confirm that $\mu_1 > 0$, $\mu_2 < 0$, and both parameters with different magnitudes ($|\mu_1| < |\mu_2|$) are economically and statistically significant.

5.5 Confronting the model with data

In implementing our structural estimation procedure, we consider that model-implied upside stock return variance, downside stock return variance, and their corresponding premia perfectly match the observed series. Thus, there are at least three remaining empirical challenges for the equilibrium model: comparing structural model-implied to observed (1) predictive regression slopes of excess return on variance risk-premium components, (2) equity return and consumption growth expectations, and (3) consumption growth variance dynamics.

Figure (2) graphically summarizes additional performance results of equilibrium model. It shows that the predictive regression slopes as per equation (12) lie within the observed 95% confidence bounds. We also see that the equilibrium model-implied conditional variance of consumption growth tracks remarkably well the EGARCH forecasts. Overall, the proposed general equilibrium model delivers empirically grounded evidence supporting its theoretical implications.

6 Conclusion

In this study, we have decomposed the celebrated variance risk premium of Bollerslev, Tauchen, and Zhou (2009) – arguably one of the most successful short-term predictors of excess equity returns –

to show that its prediction power stems from the downside variance risk premium embedded in this measure. Market participants seem more concerned with market downturns and demand a premium for bearing that risk. By contrast, they seem to like upward uncertainty in the market. We support this intuition through a simple equilibrium consumption-based asset pricing model. We develop a model where consumption growth features separate upside and downside time-varying shock processes, with feedback from volatilities to future growth. We show that under mild requirements about consumption growth and upside and downside volatility processes, we can characterize the equity premium, upside and downside variance risk premia, and the skewness risk premium that support the main stylized facts observed in our empirical investigation.

Empirically, we demonstrate that the downside variance risk premium – the difference between option-implied, risk-neutral expectations of market downside variance and historical, realized downside variances – demonstrates significant prediction power (that is at least as powerful as the variance risk premium, and often stronger) for excess returns. We also show that the difference between upside and downside variance risk premia – our proposed measure of the skewness risk premium – is both a priced factor in equity markets and a powerful predictor of excess returns. The skewness risk premium performs well for intermediate prediction steps beyond the reach of short-run predictors such as downside variance risk or variance risk premia and long-term predictors such as price-dividend or price-earning ratios alike. The skewness risk premium constructed from one month's worth of data predicts excess returns from eight months to a year ahead. The same measure constructed from one quarter's worth of data predicts monthly excess returns from four months to one year ahead. We show that our findings demonstrate remarkable robustness to the inclusion of common pricing variables. Downside variance risk and skewness risk premia have similar or better forecast ability in comparison with common predictors. Finally, while these premia are connected to macroeconomic and financial indicators, they contain useful additional information. They also markedly react to decisions or announcements that modify the uncertainty anticipated by market participants.

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Table 1: **Summary Statistics**

	Mean (%)	**Median** (%)	**Std. Dev.** (%)	**Skewness**	**Kurtosis**	**AR(1)**
Panel A: Excess Returns						
Equity	1.9771	14.5157	20.9463	-0.1531	10.5559	-0.0819
Equity (1996-2007)	3.0724	12.5824	17.6474	-0.1379	5.9656	-0.0165
Panel B: Risk-Neutral						
Variance	19.3544	18.7174	6.6110	1.5650	7.6100	0.9466
Downside Variance	16.9766	16.2104	5.8727	1.6746	8.0637	0.9548
Upside Variance	9.2570	9.1825	3.1295	1.1479	6.0030	0.8991
Skewness	-7.7196	-7.0090	3.0039	-2.0380	9.6242	0.7323
Panel C: Realized						
Variance	16.7137	15.3429	5.5216	3.6748	25.6985	0.9667
Downside Variance	11.7677	10.8670	3.9857	3.9042	29.4323	0.9603
Upside Variance	11.8550	10.8683	3.8639	3.6288	25.3706	0.9609
Skewness	0.0872	0.1315	1.0911	-6.3619	170.4998	0.6319
Panel D: Risk Premium						
Variance	2.6407	2.3932	4.2538	-0.3083	6.8325	0.9265
Downside Variance	5.2089	4.8693	3.8159	0.2019	4.6310	0.9444
Upside Variance	-2.5979	-2.5730	2.5876	-2.2178	22.7198	0.8877
Skewness	-7.8068	-6.9942	3.0606	-2.0696	10.6270	0.9345

This table reports the summary statistics for the quantities investigated in this study. Mean, median, and standard deviation values are annualized and in percentages. We report excess kurtosis values. $AR(1)$ represents the values for the first autocorrelation coefficient. The full sample is from September 1996 to December 2010. We also consider a sub-sample ending in December 2007.

Table 2: **Predictive Content of Premium Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Variance Risk Premium							
1	2.43	2.61	2.51	2.83	1.02	0.02	0.68	-0.30
2	2.84	3.76	3.42	5.58	1.50	0.68	1.04	0.05
3	4.11	8.13	3.58	6.18	1.78	1.19	1.56	0.78
6	2.78	3.65	2.24	2.22	1.57	0.82	2.09	1.87
9	1.98	1.65	1.94	1.57	1.47	0.66	1.98	1.64
12	1.96	1.64	1.43	0.61	1.53	0.77	1.73	1.14
k	Panel B: Downside Variance Risk Premium							
1	2.57	2.99	2.68	3.30	1.27	0.34	0.95	-0.06
2	3.22	4.92	4.08	7.95	2.07	1.78	1.54	0.74
3	4.76	10.72	4.46	9.50	2.61	3.12	2.32	2.37
6	3.72	6.75	3.42	5.70	2.84	3.85	3.21	4.98
9	2.96	4.27	3.14	4.86	2.82	3.86	2.99	4.35
12	3.04	4.60	2.65	3.39	2.81	3.86	2.80	3.84
k	Panel C: Upside Variance Risk Premium							
1	2.08	1.79	1.91	1.44	0.44	-0.44	-0.04	-0.55
2	2.15	1.96	2.15	1.96	0.39	-0.47	-0.18	-0.54
3	3.05	4.41	2.07	1.79	0.26	-0.52	-0.27	-0.52
6	1.57	0.82	0.61	-0.36	-0.40	-0.48	-0.27	-0.53
9	0.83	-0.18	0.36	-0.50	-0.52	-0.42	-0.14	-0.57
12	0.74	-0.27	-0.05	-0.59	-0.33	-0.52	-0.41	-0.49
k	Panel D: Skewness Risk Premium							
1	-0.10	-0.55	0.41	-0.46	0.96	-0.04	1.25	0.30
2	0.61	-0.35	1.67	0.98	1.98	1.59	2.16	1.98
3	1.03	0.04	2.24	2.18	2.81	3.70	3.29	5.17
6	2.27	2.30	3.33	5.38	4.05	8.00	4.45	9.59
9	2.57	3.13	3.39	5.70	4.20	8.73	3.98	10.59
12	2.83	3.95	3.43	5.93	3.88	7.60	4.07	8.34

This table reports predictive regression results for prediction horizons (k) between 1 and 12 months ahead, and aggregation levels (h) between 1 and 12 months, based on a predictive regression model of the form $r_{t\to t+k} = \beta_0 + \beta_1 x_t(h) + \varepsilon_{t\to t+k}$. In this regression model, $r_{t\to t+k}$ is the cumulative excess returns between t and $t+k$; $x_t(h)$ is the proposed variance or skewness risk premia component that takes the values from variance risk, upside variance risk, downside variance risk, or skewness risk premia measures; and $\varepsilon_{t\to t+k}$ is a zero-mean error term. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 3: **Predictive Content of Risk-Neutral Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Risk-Neutral Variance							
1	0.28	-0.51	0.50	-0.41	0.69	-0.29	0.75	-0.24
2	1.14	0.17	1.24	0.30	1.35	0.44	1.39	0.51
3	1.30	0.38	1.52	0.72	1.83	1.28	2.13	1.92
6	2.10	1.88	2.33	2.43	2.76	3.57	3.21	4.95
9	2.32	2.44	2.55	3.05	2.95	4.22	3.15	4.85
12	2.21	2.20	2.45	2.82	2.89	4.11	3.30	5.45
k	Panel B: Risk-Neutral Downside Variance							
1	0.27	-0.51	0.57	-0.37	0.77	-0.23	0.87	-0.14
2	1.22	0.27	1.39	0.51	1.49	0.66	1.54	0.74
3	1.42	0.56	1.70	1.04	2.03	1.70	2.35	2.44
6	2.23	2.17	2.52	2.91	2.97	4.21	3.42	5.67
9	2.43	2.71	2.68	3.42	3.10	4.70	3.26	5.22
12	2.32	2.48	2.55	3.09	2.99	4.42	3.42	5.84
k	Panel C: Risk-Neutral Upside Variance							
1	0.29	-0.50	0.27	-0.51	0.36	-0.48	0.20	-0.53
2	0.93	-0.07	0.76	-0.23	0.78	-0.21	0.60	-0.35
3	0.99	-0.02	0.94	-0.06	1.04	0.05	1.00	0.00
6	1.74	1.12	1.72	1.09	1.92	1.48	2.05	1.77
9	2.01	1.71	2.09	1.89	2.31	2.42	2.38	2.59
12	1.90	1.49	2.09	1.92	2.45	2.83	2.57	3.17
k	Panel D: Risk-Neutral Skewness							
1	0.22	-0.52	0.87	-0.14	1.10	0.11	1.27	0.34
2	1.51	0.70	2.02	1.67	2.06	1.76	2.08	1.79
3	1.93	1.48	2.47	2.74	2.85	3.80	3.13	4.65
6	2.70	3.42	3.28	5.21	3.80	7.02	4.11	8.18
9	2.76	3.64	3.17	4.92	3.64	6.54	3.56	6.26
12	2.67	3.44	2.88	4.07	3.27	5.35	3.66	6.73

This table reports predictive regression results for risk-neutral variance and skewness measures. The predictive regression model, prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 2. The difference is in the definition of $x_t(h)$: Instead of risk premia, we use risk-neutral measures for variance, upside variance, downside variance, and skewness. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 4: **Predictive Content of Realized (Physical) Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Realized Variance							
1	-1.10	0.12	-0.99	-0.01	-0.10	-0.55	0.09	-0.55
2	-0.67	-0.30	-0.86	-0.15	0.18	-0.54	0.40	-0.47
3	-1.18	0.22	-0.75	-0.25	0.36	-0.48	0.62	-0.34
6	0.01	-0.57	0.48	-0.44	1.13	0.15	1.07	0.09
9	0.55	-0.40	0.78	-0.22	1.33	0.44	1.12	0.15
12	0.49	-0.45	0.98	-0.02	1.27	0.35	1.40	0.56
k	Panel B: Realized Downside Variance							
1	-1.05	0.06	-0.90	-0.10	-0.08	-0.55	0.09	-0.55
2	-0.53	-0.40	-0.76	-0.23	0.21	-0.53	0.39	-0.47
3	-1.04	0.05	-0.68	-0.30	0.39	-0.48	0.59	-0.36
6	0.05	-0.57	0.48	-0.44	1.08	0.10	0.99	-0.01
9	0.54	-0.41	0.75	-0.25	1.21	0.27	1.01	0.01
12	0.44	-0.48	0.89	-0.12	1.14	0.18	1.30	0.40
k	Panel C: Realized Upside Variance							
1	-1.15	0.18	-1.09	0.10	-0.13	-0.54	0.10	-0.55
2	-0.82	-0.18	-0.95	-0.05	0.14	-0.54	0.41	-0.46
3	-1.33	0.43	-0.82	-0.18	0.34	-0.49	0.66	-0.32
6	-0.05	-0.57	0.48	-0.44	1.17	0.21	1.16	0.19
9	0.39	-0.41	0.81	-0.19	1.44	0.61	1.23	0.29
12	0.54	-0.42	1.08	0.09	1.39	0.54	1.51	0.74
k	Panel D: Realized Skewness							
1	0.44	-0.45	1.58	0.81	0.63	-0.33	-0.06	-0.55
2	1.51	0.70	1.67	0.99	0.96	-0.05	-0.26	-0.52
3	1.45	0.61	1.19	0.23	0.71	-0.28	-0.89	-0.12
6	0.54	-0.40	0.11	-0.56	-1.01	0.01	-2.64	3.25
9	0.07	-0.58	-0.54	-0.41	-3.01	4.41	-3.67	6.67
12	-0.55	-0.41	-1.82	1.33	-3.37	5.70	-3.36	5.67

This table reports predictive regression results for realized variance and skewness measures. The predictive regression model, prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 2. The difference is in the definition of $x_t(h)$: Instead of risk premia, we use realized (historical) measures for variance, upside variance, downside variance, and skewness. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 5: **Joint Regression Results**

h	1			3			6			12		
	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$
k	Up	Down		Up	Down		Up	Down		Up	Down	
Panel A: Risk Premium												
1	-0.01	1.49	2.45	-0.12	1.86	2.77	-0.58	1.32	-0.03	-0.85	1.27	-0.21
2	-0.78	2.49	4.72	-1.28	3.66	8.28	-1.38	2.46	2.26	-1.54	2.17	1.49
3	-1.28	3.79	11.04	-1.81	4.32	10.63	-2.06	3.33	4.84	-2.34	3.31	4.77
6	-2.46	4.19	9.36	-3.00	4.56	9.80	-3.31	4.39	8.98	-3.14	4.54	9.54
9	-2.75	3.99	7.76	-3.10	4.45	9.36	-3.46	4.49	9.50	-2.74	4.09	7.83
12	-3.06	4.29	9.06	-3.18	4.18	8.30	-3.13	4.24	8.61	-2.97	4.10	8.06
Panel B: Risk-Neutral Measures												
1	0.08	-0.01	-1.06	-1.13	1.24	-0.22	-1.30	1.46	0.15	-1.48	1.70	0.51
2	-1.00	1.27	0.27	-2.42	2.69	3.10	-2.27	2.61	2.90	-2.00	2.45	2.35
3	-1.59	1.89	1.39	-2.92	3.26	5.01	-3.22	3.68	6.55	-2.87	3.59	6.21
6	-1.64	2.14	3.09	-2.93	3.47	6.89	-3.25	3.99	9.12	-2.61	3.79	8.66
9	-1.32	1.88	3.12	-2.02	2.62	5.09	-2.25	3.05	6.88	-1.42	2.62	5.78
12	-1.35	1.89	2.94	-1.49	2.07	3.77	-1.39	2.18	4.93	-1.28	2.55	6.20
Panel C: Realized (Physical) Measures												
1	-0.63	0.42	-0.28	-1.82	1.72	1.17	-0.69	0.68	-0.84	0.11	-0.10	-1.10
2	-1.62	1.50	0.51	-1.92	1.83	1.24	-0.93	0.95	-0.60	0.48	-0.46	-0.90
3	-1.68	1.46	1.05	-1.41	1.34	0.25	-0.62	0.64	-0.82	1.27	-1.24	-0.02
6	-0.54	0.54	-0.97	-0.01	0.06	-1.01	1.39	-1.31	0.61	3.54	-3.49	6.14
9	0.01	0.08	-0.99	0.72	-0.64	-0.54	3.61	-3.52	6.77	4.82	-4.76	11.39
12	0.63	-0.55	-0.83	2.07	-1.98	1.77	3.99	-3.91	8.24	4.66	-4.59	11.22

This table reports predictive regression results when multiple variance components (risk premia, risk-neutral, and realized measures) are included in the regression model. The prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 2. The difference is in the regression model. Both upside and downside variance components are in the model: $r_{t\to t+k} = \beta_0 + \beta_1 x_{1,t}(h) + \beta_2 x_{2,t}(h) + \varepsilon_{t\to t+k}$. $x_{1,t}(h)$ pertains to upside measures and $x_{2,t}(h)$ represents the downside measures used in the analysis. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 6: **Semi-Annual Simple Predictive Regressions, September 1996 to December 2010**

Intercept	0.0005	-0.0026	-0.0132	0.0012	0.0756	0.0861	0.0514	0.0004	-0.0000	0.0210	-0.0105	-0.0093
	(0.1995)	(-1.1632)	(-3.0304)	(0.7230)	(2.9076)	(3.1978)	(2.0974)	(0.1293)	(-0.0067)	(6.3627)	(-3.1169)	(-2.0903)
$uvrp_t$	-0.0179											
	(-0.3917)											
$dvrp_t$		0.1135										
		(2.5900)										
srp_t			-0.1838									
			(-3.6007)									
vrp_t				0.0516								
				(1.4937)								
$log(p_t/d_t)$					-0.0419							
					(-2.8630)							
$log(p_{t-1}/d_t)$						-0.0478						
						(-3.1550)						
$log(p_t/e_t)$							-0.0384					
							(-2.0487)					
tms_t								0.7181				
								(0.4233)				
dfs_t									1.6755			
									(0.3885)			
$infl_t$										-0.8052		
										(-6.7137)		
$kpis_t$											0.1472	
											(4.0170)	
$kpos_t$												0.1203
												(2.5798)
Adj. R^2(%)	-0.5157	3.3439	6.7611	0.7406	4.1793	5.1472	1.9009	-0.5000	-0.5172	21.0807	8.4028	3.3140

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on each one-period (1-month) lagged predictor from September 1996 to December 2010. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 7: **Semi-Annual Multiple Predictive Regressions, September 1996 to December 2010**

Intercept	-0.0128	-0.0128	-0.0133	0.0787	0.0951	0.0582	-0.0045	-0.0084	0.0171	-0.0137	-0.0131
	(-2.9375)	(-2.9375)	(-2.9489)	(3.0953)	(3.6144)	(2.4206)	(-1.3553)	(-1.7752)	(4.9989)	(-3.8625)	(-2.8590)
$uvrp_t$	-0.1545										
	(-2.7123)										
$dvrp_t$	0.2100	0.0555	0.4321	0.1272	0.1392	0.1309	0.1178	0.1359	0.1289	0.1048	0.1129
	(3.7629)	(1.1549)	(3.4605)	(2.9688)	(3.2550)	(2.9980)	(2.6638)	(2.9172)	(3.3499)	(2.4918)	(2.6228)
srp_t		-0.1545									
		(-2.7123)									
vrp_t			-0.2640								
			(-2.7177)								
$log(p_t/d_t)$				-0.0462							
				(-3.2112)							
$log(p_{t-1}/d_t)$					-0.0557						
					(-3.7277)						
$log(p_t/e_t)$						-0.0471					
						(-2.5413)					
tms_t							1.2900				
							(0.7681)				
dfs_t								6.2346			
								(1.3864)			
$infl_t$									-0.8272		
									(-7.0977)		
$kpis_t$										0.1425	
										(3.9439)	
$kpos_t$											0.1197
											(2.6128)
Adj. R^2(%)	6.9505	6.9505	6.9664	8.5372	10.3900	6.4572	3.1016	3.8843	25.7111	11.2225	6.6602

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on one-period (1-month) lagged downside variance risk premium $dvrp$ and one alternative predictor in turn from September 1996 to December 2010. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 8: **Semi-Annual Simple Predictive Regressions, September 1996 to December 2007**

Intercept	0.0129	-0.0070	-0.0124	0.0014	0.2016	0.2164	0.0913	0.0035	0.0206	0.0052	-0.0059	-0.0047
	(5.1388)	(-3.5015)	(-2.6470)	(1.0794)	(6.6115)	(7.0123)	(3.9851)	(1.5568)	(3.4844)	(1.0608)	(-2.2330)	(-1.3202)
$uvrp_t$	0.2679											
	(4.5558)											
$dvrp_t$		0.2784										
		(6.6863)										
srp_t			-0.2156									
			(-3.5176)									
vrp_t				0.2300								
				(6.5106)								
$log(p_t/d_t)$					-0.1092							
					(-6.5085)							
$log(p_{t-1}/d_t)$						-0.1176						
						(-6.9106)						
$log(p_t/e_t)$							-0.0662					
							(-3.8474)					
tms_t								-0.1990				
								(-0.1295)				
dfs_t									-25.6836			
									(-3.0118)			
$infl_t$										-0.0735		
										(-0.4029)		
$kpis_t$											0.1217	
											(4.0782)	
$kpos_t$												0.0940
												(2.4679)
Adj. R^2(%)	13.2802	25.3069	8.1025	24.2902	24.2781	26.6030	9.6656	-0.7680	5.8882	-0.6536	10.8080	3.7964

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on each one-period (1-month) lagged predictor from September 1996 to December 2007. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 9: **Semi-Annual Multiple Predictive Regressions, September 1996 to December 2007**

Intercept	-0.0045	-0.0045	-0.0049	0.1736	0.1954	0.1053	-0.0087	-0.0033	-0.0142	-0.0155	-0.0162
	(-1.0099)	(-1.0099)	(-1.0492)	(6.6158)	(7.5813)	(5.7055)	(-3.2688)	(-0.4896)	(-2.8139)	(-5.8453)	(-4.7195)
$uvrp_t$	0.0454										
	(0.6200)										
$dvrp_t$	0.2554	0.3008	0.2065	0.2534	0.2629	0.3156	0.2853	0.2669	0.2980	0.2716	0.2847
	(4.5711)	(5.4521)	(1.4127)	(7.0696)	(7.6580)	(8.4728)	(6.7545)	(5.7833)	(6.8848)	(6.9931)	(7.0767)
srp_t		0.0454									
		(0.6200)									
vrp_t			0.0632								
			(0.5133)								
$log(p_t/d_t)$				-0.0990							
				(-6.8974)							
$log(p_{t-1}/d_t)$					-0.1113						
					(-7.8692)						
$log(p_t/e_t)$						-0.0855					
						(-6.1130)					
tms_t							1.3106				
							(0.9772)				
dfs_t								-4.9164			
								(-0.5838)			
$infl_t$									0.2541		
									(1.5553)		
$kpis_t$										0.1148	
										(4.5066)	
$kpos_t$											0.1050
											(3.2382)
Adj. R^2(%)	24.9460	24.9460	24.8746	45.2343	49.3937	41.8336	25.2805	24.9203	26.1258	35.0975	30.4605

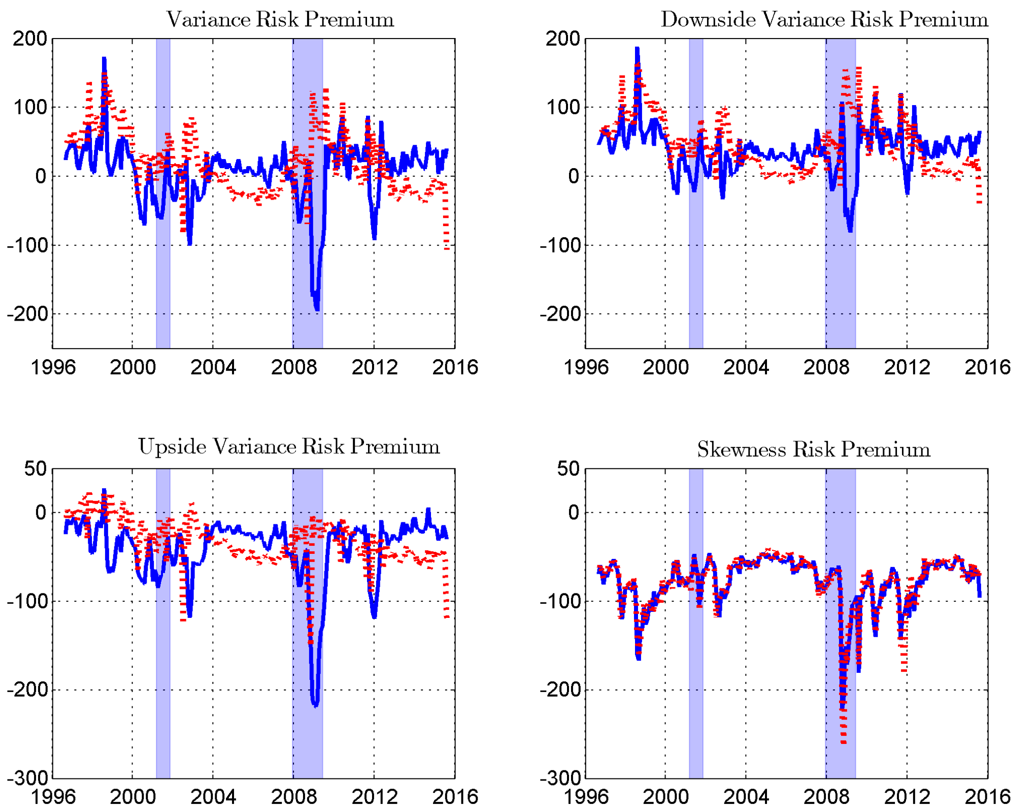
This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on one-period (1-month) lagged downside variance risk premium $dvrp$ and one alternative predictor in turn from September 1996 to December 2007. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 10: **Structural Estimation of the Theoretical Model**

Parameters	Estimates	Std. Err.
γ	1.01	0.00
θ	-0.04	0.02
δ	1.00	0.00
μ_0	0.32	0.10
μ_1	0.05	0.01
μ_2	-0.10	0.01
σ_c	0.87	0.04
α_u	0.30	0.03
β_u	0.99	0.00
α_d	0.16	0.01
β_d	0.99	0.00
$\gamma_{u,0}$	8216.00	3074.52
$\gamma_{u,1}$	0.53	0.00
φ_u	113920.58	47206.21
$\gamma_{d,0}$	1267.55	201.23
$\gamma_{d,1}$	0.55	0.00
φ_d	47552.57	17481.63
κ_0	0.00	0.00
κ_1	0.13	0.04

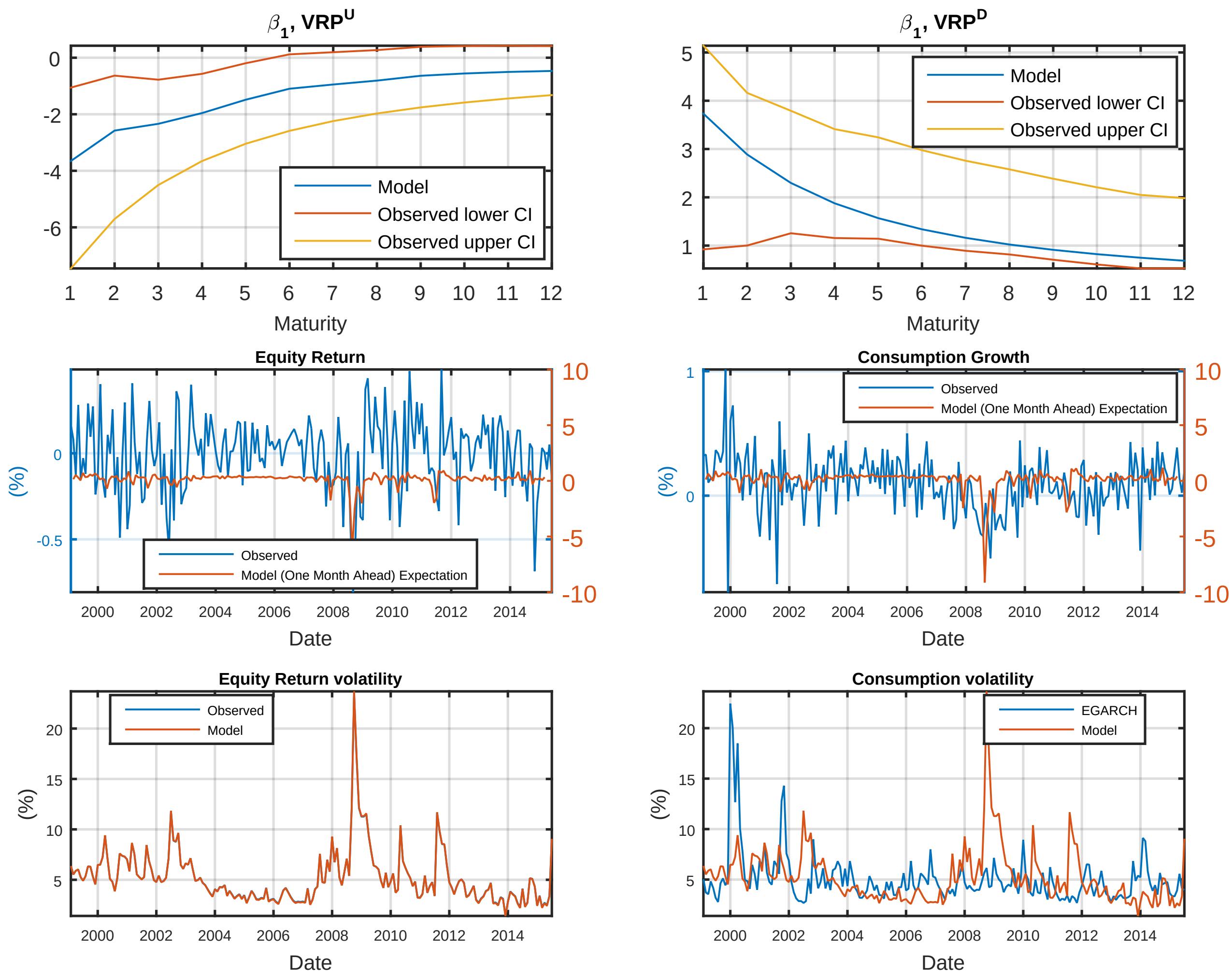
This table reports the structural parameter estimates of the general equilibrium model along with their standard errors (Std. Err.). These estimates are obtained by maximizing the joint likelihood of consumption growth, stock return and risk-free rate.

Figure 1: Time Series for Variance and Skewness Risk Premia



These figures plot the paths of annualized monthly values ($\times 10^3$) for the variance risk premium, upside variance risk premium, downside variance risk premium, and skewness risk premium, extracted from U.S. financial markets data for September 1996 to March 2015. Solid lines represent premia constructed from random walk forecasts of the realized volatility and components. The dotted lines represent the same for M-HAR forecasts of the realized volatility and components. The shaded areas represent NBER recessions.

Figure 2: Confronting the General Equilibrium Model with Data



The top plots show the slopes of the regressions (as per equation (12)) of excess return on upside (resp. downside) variance risk premium implied by the general equilibrium model at different maturities (expressed in months), along with the observed 95% confidence intervals. The middle (resp. bottom) plots present monthly model-implied equity return and consumption growth (resp. volatility) paths against the observed corresponding series.